

SU(3) Einstein-Skyrme Solitons and Black Holes

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March 13, 1995

Abstract

In the SU(3) Einstein-Skyrme system static spherically symmetric particle-like solutions and black holes exist for both the SU(2) and the SO(3) embedding. The SO(3) embedding leads to new particle-like solutions and black holes, sharing many features with the SU(2) solutions. In particular, there are always two branches of solutions, forming a cusp at a critical coupling constant. The regular SO(3) solutions have even topological charge B . The mass of the $B = 2$ SO(3) solutions is less than twice the mass of the $B = 1$ SU(2) solutions. We conjecture, that the lowest SO(3) branches correspond to stable particle-like solutions and stable black holes.

Utrecht-Preprint THU-95/6; hep-th/9503087

1 Introduction

The SU(2) Einstein-Skyrme system possesses two different types of black hole solutions with the same mass for a certain range of coupling constants [1, 2, 3]. These are the Schwarzschild black holes with vanishing chiral fields and the SU(2) Einstein-Skyrme black holes with “chiral hair”. For a given event horizon, there are always two branches of black hole solutions with “chiral hair” which form a cusp at a critical coupling constant (dependent on the event horizon) [3]. The black hole solutions on the lower branches are linearly stable [4, 3]. Thus for a certain range of masses the system possesses two distinct classically stable types of black holes, providing a counterexample to the “no-hair conjecture” for black holes.

Beside the black hole solutions the SU(2) Einstein-Skyrme system also possesses two branches of regular solutions, which merge at a critical coupling constant [3]. Again, the solutions on the lower branch are classically stable [3, 4], they correspond to solitons. For zero coupling the lower branch starts from the SU(2) skyrmion solution in flat space [5, 6], while the upper branch ends in the lowest SU(2) Einstein-Yang-Mills sphaleron solution [7] (after scaling the radial coordinate [3]).

Here we consider particle-like solutions and black holes in the SU(3) Einstein-Skyrme system. In the SU(3) Skyrme model in flat space regular static spherically symmetric solutions exist for both subgroups SU(2) and SO(3) [8]. The SU(2) embedding yields as lowest solution the stable SU(2) skyrmion with topological number $B = 1$. The SO(3) embedding yields as lowest solution a stable solution with topological number $B = 2$, whose mass is lower than twice the mass of the SU(2) skyrmion [8]. These SO(3) solutions are recovered in the SU(3) Einstein-Skyrme system for vanishing coupling. They form the starting point of our investigation. We then vary the coupling constant to construct the whole family of particle-like SO(3) solutions, consisting of a lower and an upper branch. As endpoint on the upper branch we obtain an SO(3) sphaleron solution (after scaling the radial coordinate).

Analogous to the SU(2) Einstein-Skyrme system, asymptotically flat SO(3) black hole solutions emerge from the regular solutions by requiring regularity at a finite event horizon. For a given horizon, two branches of SO(3) black holes with “chiral hair” exist below a critical coupling constant (dependent on the horizon). We argue, that the SO(3) black hole solutions on the lower branch are classically stable.

2 SU(3) Einstein-Skyrme Equations of Motion

We consider the SU(3) Einstein-Skyrme action

$$S = S_G + S_M = \int L_G \sqrt{-g} d^4x + \int L_M \sqrt{-g} d^4x \quad (1)$$

with

$$L_G = \frac{1}{16\pi G} R , \quad (2)$$

and the matter Lagrangian for the chiral SU(3) field U

$$L_M = \frac{1}{4} f^2 \text{Tr}(A_\mu A^\mu) + \frac{1}{32e^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) , \quad (3)$$

where

$$A_\mu = U^\dagger \nabla_\mu U = U^\dagger \partial_\mu U \quad (4)$$

and

$$F_{\mu\nu} = [A_\mu, A_\nu] , \quad (5)$$

and f and e are coupling constants. Variation of the action eq. (1) with respect to the metric $g_{\mu\nu}$ and the chiral field U leads to the Einstein equations and the matter field equations.

To construct static spherically symmetric solitons and black holes we employ Schwarzschild-like coordinates and adopt the spherically symmetric metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -A^2 N dt^2 + N^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) , \quad (6)$$

with

$$N = 1 - \frac{2m}{r} . \quad (7)$$

Generalized spherical symmetry for the chiral field is realized by embedding the SU(2) or the SO(3) generators T_i in SU(3). In the SU(2)-embedding $\vec{T} = \frac{1}{2}(\lambda_1, \lambda_2, \lambda_3)$, and the ansatz for the chiral field is

$$U = \begin{pmatrix} \cos \xi + i\tau_r \sin \xi & 0 \\ 0 & 1 \end{pmatrix} , \quad (8)$$

with the SU(2) Pauli matrices τ_i and $\tau_r = (\tau_1, \tau_2, \tau_3) \cdot \vec{e}_r$. (The general ansatz for U includes a factor of $\exp(i\lambda_8 \zeta)$, however $\zeta = 0$ for the lowest solution [8].) In the SO(3)-embedding $\vec{T} = (\lambda_7, -\lambda_5, \lambda_2)$, and the ansatz for the chiral field is

$$U = \exp\left(i\Lambda_r \chi + i\left(\Lambda_r^2 - \frac{2}{3}\right)\phi\right) \quad (9)$$

with $\Lambda_r = (\lambda_7, -\lambda_5, \lambda_2) \cdot \vec{e}_r$. An equivalent expression for the chiral field is

$$U = \exp(i\psi) + i\Lambda_r \exp(-i\frac{\psi}{2}) \sin \chi + \Lambda_r^2 \left(\exp(-i\frac{\psi}{2}) \cos \chi - \exp(i\psi) \right) , \quad (10)$$

with $\phi = -3\psi/2$.

The SU(2)-embedding, eq. (8), leads to the well studied SU(2) Einstein-Skyrme equations [2, 3]. To obtain the SU(3) Einstein-Skyrme equations for the SO(3)-embedding, eq. (9), we also employ the tt and rr components of the Einstein equations, yielding

$$\begin{aligned}\mu' = & \alpha \left(\frac{1}{2} x^2 N (\chi'^2 + \frac{1}{3} \phi'^2) + 2(1 - \cos \phi \cos \chi) \right. \\ & + \frac{1}{2} N (1 - \cos \phi \cos \chi) (\chi'^2 + \phi'^2) + N \sin \phi \sin \chi \phi' \chi' \\ & \left. + \frac{3}{2x^2} \sin^2 \phi \sin^2 \chi + \frac{1}{2x^2} (1 - \cos \phi \cos \chi)^2 \right),\end{aligned}\quad (11)$$

and

$$\begin{aligned}A' = & \alpha \left(x \left(\chi'^2 + \frac{1}{3} \phi'^2 \right) \right. \\ & \left. + \frac{1}{x} \left((1 - \cos \phi \cos \chi) (\chi'^2 + \phi'^2) + 2 \sin \phi \sin \chi \phi' \chi' \right) \right) A,\end{aligned}\quad (12)$$

where we have introduced the dimensionless mass function

$$\mu = e f m \quad (13)$$

and the dimensionless coordinate $x = e f r$, the prime indicates the derivative with respect to x , and

$$\alpha = 4\pi G f^2 = 4\pi \left(\frac{f}{m_{\text{Pl}}} \right)^2 \quad (14)$$

is a dimensionless coupling constant. For the matter functions we obtain the equations

$$\begin{aligned}& \left(AN \left((1 + x^2 - \cos \phi \cos \chi) \chi' + \sin \phi \sin \chi \phi' \right) \right)' \\ & = A \left(\cos \phi \sin \chi \left(2 + \frac{N}{2} (\chi'^2 + \phi'^2) + \frac{1}{x^2} (1 - \cos \phi \cos \chi) \right) \right. \\ & \quad \left. + \sin \phi \cos \chi \left(N \phi' \chi' + \frac{3}{x^2} \sin \phi \sin \chi \right) \right),\end{aligned}\quad (15)$$

and

$$\begin{aligned}& \left(AN \left(\left(1 + \frac{1}{3} x^2 - \cos \phi \cos \chi \right) \phi' + \sin \phi \sin \chi \chi' \right) \right)' \\ & = A \left(\sin \phi \cos \chi \left(2 + \frac{N}{2} (\chi'^2 + \phi'^2) + \frac{1}{x^2} (1 - \cos \phi \cos \chi) \right) \right. \\ & \quad \left. + \cos \phi \sin \chi \left(N \phi' \chi' + \frac{3}{x^2} \sin \phi \sin \chi \right) \right).\end{aligned}\quad (16)$$

With help of eq. (12) the metric function A can be eliminated from the matter equations.

The Einstein-Skyrme system has a topological current

$$B^\mu = -\frac{1}{\sqrt{-g}} \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr} A_\nu A_\alpha A_\beta \quad (17)$$

which is covariantly conserved, yielding the topological charge

$$B = \int \sqrt{-g} B^0 d^3x = -\frac{1}{2\pi^2} \int \left((1 - \cos \phi \cos \chi) \chi' + \sin \phi \sin \chi \phi' \right) dx \sin \theta d\theta d\phi . \quad (18)$$

3 Regular Solutions

Let us consider the regular particle-like solutions of the SU(3) Einstein-Skyrme system. Requiring asymptotically flat solutions implies that the metric functions A and μ both approach a constant at infinity, and that the chiral functions approach an integer multiple of π . We here adopt

$$A(\infty) = 1 , \quad (19)$$

thus fixing the time coordinate, and

$$\chi(\infty) = 0 , \quad \phi(\infty) = 0 . \quad (20)$$

At the origin regularity of the solutions requires

$$\mu(0) = 0 , \quad (21)$$

and that the chiral functions again approach an integer multiple of π . The lowest non-trivial solution has [8]

$$\chi(0) = \pi , \quad \phi(0) = \pi , \quad (22)$$

and the choice $\chi(0) = \pi$, $\phi(0) = -\pi$ leads to a degenerate solution. The boundary conditions

$$\chi(0) = 2\pi , \quad \phi(0) = 0 , \quad (23)$$

lead to the known SU(2) solutions corresponding to the modified coupling constant $\tilde{\alpha} = 4\alpha$ with $\chi = 2\xi$ and $\phi = 0$. (Thus for the boundary conditions (23) the SO(3) masses for the coupling constant α are four times as big as the SU(2) masses for the coupling constant $\tilde{\alpha}$, and the critical SO(3) coupling constant is one fourth of the critical SU(2) coupling constant.)

The topological charge of the solutions [8]

$$B = -\frac{2}{\pi} (\chi - \sin \chi \cos \phi) \Big|_0^\infty \quad (24)$$

depends only on the boundary conditions. For the boundary conditions (22) and (23) the topological charge is $B = 2$ and $B = 4$, respectively. In the following we present the numerical solutions for the SO(3) embedding with boundary conditions (22), corresponding to $B = 2$ solutions, since the solutions of the SU(2) embedding reproduce the known SU(2) results.

The regular SO(3) Einstein-Skyrme solutions and the regular SU(2) Einstein-Skyrme solutions have many features in common. The equations depend only on the coupling constant $\alpha = 4\pi G f^2$, whose variation can be considered in two different ways, either as changing G and keeping f constant, or vice versa. This results in the occurrence of two distinct branches of solutions, which exist for a finite range of coupling constants $[0, \alpha_c]$. The branches merge in a cusp at α_c and beyond this critical coupling no solutions exist. The value of α_c depends on the embedding and on the boundary conditions.

In Fig. 1 we demonstrate this feature by presenting the mass of the $B = 1$ SU(2) solutions and the $B = 2$ SO(3) solutions along both branches. The mass of the SO(3) solutions is always lower than twice the mass of the SU(2) solutions. The ADM mass

$$m_{\text{ADM}} = \mu(\infty) \frac{4\pi}{\alpha} \frac{f}{e} \quad (25)$$

can be read off the figure, and the critical SO(3) coupling is $\alpha_c = 0.02728$, while the critical SU(2) coupling is $\alpha_c = 0.04038$ [2, 3].

Considering the limit $\alpha \rightarrow 0$, the solutions on the lower branch reduce to those of the Skyrme model in flat space with $\mu = 0$. We therefore refer to this branch as the skyrmion branch. In the limit $\alpha \rightarrow 0$ the solutions on the upper branch shrink to zero size, while their mass diverges. This is, however, only due to the particular choice of variables, as observed by Bizon and Chmaj [3] for SU(2). With the coordinate transformation $\tilde{x} = x/\sqrt{\alpha}$ [3] the equations for the metric and matter functions become identical to those of the static spherically symmetric magnetic Einstein-Yang-Mills system in the limit $\alpha \rightarrow 0$, where sphaleron solutions exist [9, 10]. We therefore refer to the upper branch as sphaleron branch. Here we note only, that the mass of the lowest SO(3) sphaleron is $\tilde{\mu}(\infty) = 1.3078$ [10], while the mass of the lowest SU(2) sphaleron [7] is $\tilde{\mu}(\infty) = 0.8286$, and $m_{\text{ADM}} = \tilde{\mu}(\infty)\sqrt{4\pi}m_{\text{Pl}}/e$.

Let us now consider the family of regular SO(3) solutions obtained by first increasing α along the skyrmion branch from zero up to the critical coupling α_c , and then decreasing α again along the sphaleron branch back to zero. Starting from the SO(3) skyrmion solution in flat space, the regular solutions change continuously, and reach finally the SO(3) sphaleron solution [10], where they shrink to zero size (with respect to the coordinate x). Along this path, the solutions can be characterized, for instance, by the derivative of χ at the origin, which changes monotonically. As an example we show the radial functions for the regular SO(3) solution along the skyrmion branch for $\alpha = 0.001$ in Figs. (2)-(5). Further details will be given elsewhere [10].

4 Black Hole Solutions

We now turn to the black hole solutions of the SU(3) Einstein-Skyrme system. Imposing again the condition of asymptotic flatness, the black hole solutions satisfy the same boundary conditions at infinity as the regular solutions. The existence of a regular event horizon at x_H requires

$$\mu(x_H) = \frac{x_H}{2} , \quad (26)$$

and $A(x_H) < \infty$, and the chiral functions must satisfy at the horizon x_H

$$\begin{aligned} & N' \left((1 + x^2 - \cos \phi \cos \chi) \chi' + \sin \phi \sin \chi \phi' \right) \\ &= \cos \phi \sin \chi \left(2 + \frac{1}{x^2} (1 - \cos \phi \cos \chi) \right) + \frac{3}{2x^2} \sin^2 \phi \sin 2\chi , \end{aligned} \quad (27)$$

$$\begin{aligned} & N' \left(\left(1 + \frac{1}{3} x^2 - \cos \phi \cos \chi \right) \phi' + \sin \phi \sin \chi \chi' \right) \\ &= \sin \phi \cos \chi \left(2 + \frac{1}{x^2} (1 - \cos \phi \cos \chi) \right) + \frac{3}{2x^2} \sin 2\phi \sin^2 \chi . \end{aligned} \quad (28)$$

Again, the SO(3) Einstein-Skyrme black holes and the SU(2) Einstein-Skyrme black holes have many features in common. For a given horizon x_H there are two distinct branches of solutions, which exist for a finite range of coupling constants $[0, \alpha_c(x_H)]$. The critical coupling now depends on the horizon x_H . In Fig. 6 we exhibit the masses of the SO(3) black holes in terms of the mass fractions outside the horizon, μ_{out} , defined via

$$m_{\text{ADM}} = \left(\frac{x_H}{2} + \mu_{\text{out}} \right) \frac{4\pi}{\alpha} \frac{f}{e} = \mu(\infty) \frac{4\pi}{\alpha} \frac{f}{e} . \quad (29)$$

For $x_H \rightarrow 0$ the black hole solutions approach the regular solutions. With increasing horizon x_H the critical coupling $\alpha_c(x_H)$ decreases from the critical value of the regular solutions α_c .

The families of SO(3) black hole solutions for various values of the horizon x_H change continuously, when α is increased from zero along the skyrmion branch up to the critical coupling $\alpha_c(x_H)$, and then decreased back to zero along the sphaleron branch. At zero coupling on the skyrmion branch the solutions correspond to Schwarzschild black holes with “chiral hair”. As examples we show the radial functions for the SO(3) black hole solutions with the horizons $x_H = 0.05, 0.10, 0.15$ and 0.20 along the skyrmion branch for $\alpha = 0.001$ in Figs. (2)-(5). Further details of these solutions as well as the SO(3) Einstein-Yang-Mills black holes will be given elsewhere [10].

5 Conclusion

Like the regular SU(2) solutions with topological number $B = 1$, the regular SO(3) solutions with topological number $B = 2$ form two branches, a skyrmion branch and a

sphaleron branch, existing below a critical coupling. The corresponding critical SU(2) and SO(3) couplings are $\alpha_c = 0.04038$ [2, 3] and $\alpha_c = 0.02728$, respectively. (Thus the critical coupling of the $B=2$ solutions in the SO(3) embedding is considerably larger than that of the $B=2$ solutions in the SU(2) embedding [3].) For vanishing coupling, the lower branch of the regular SO(3) solutions starts at the $B = 2$ skyrmion in flat space [8], and the upper branch ends in the lowest SO(3) Einstein-Yang-Mills sphaleron [9, 10] (after scaling the radial coordinate).

For a given coupling constant, on both branches the mass of the regular SO(3) solution with $B = 2$ is smaller than twice the mass of the SU(2) solution with $B = 1$. The $B = 1$ SU(2) and the $B = 2$ SO(3) skyrmion solutions in flat space are stable [8], and the full SU(2) skyrmion branch is stable [4, 3]. We conclude, that the full SO(3) skyrmion branch is also stable.

General arguments from catastrophe theory imply a change of stability at the critical coupling constant. (This fact is also known for skyrmions coupled to vector bosons in flat space [11, 12, 13].) For the SU(2) case it was shown explicitly, that one eigenvalue passes zero at the critical coupling [3]. The SU(2) solutions on the sphaleron branch then possess one unstable mode [3]. (Note, that the lowest SU(2) Einstein-Yang-Mills sphaleron, has two unstable modes [14, 15, 16].) Therefore we argue, that a change of stability takes place at the cusp for the SO(3) solutions as well. The SO(3) solutions on the sphaleron branch then should possess one unstable mode.

For a given value of the event horizon, the SO(3) black hole solutions also form two branches, a skyrmion branch and a sphaleron branch, with a bifurcation point at a critical coupling. This critical coupling depends on the horizon, and decreases with increasing horizon. The $B = 1$ SU(2) skyrmion branch is known to be stable, while the sphaleron branch is unstable with one unstable mode [4, 3]. Invoking arguments from catastrophe theory again [17], we conjecture, that the $B = 2$ SO(3) skyrmion branch is stable, too, while the sphaleron branch is unstable with one unstable mode, acquired at the critical coupling. Since the SO(3) Einstein-Skyrme system also contains the Schwarzschild black holes, there are then for a certain range of coupling constants (depending on the event horizon), three black hole solutions, two of which are stable: the Schwarzschild solution and the black hole solution with “chiral hair” on the skyrmion branch. This then provides an additional counterexample to the “no-hair conjecture”.

The SU(2) Einstein-Skyrme model has a rich spectrum of particle-like solutions and black holes [3], on the one hand corresponding to solutions in higher topological sectors and on the other hand corresponding to solutions related to the excited SU(2) Einstein-Yang-Mills sphalerons [7] and black holes [18, 19]. For the SO(3) Einstein-Skyrme system we expect a similarly rich spectrum, which is presently under investigation. Solutions in higher topological sectors certainly exist, and so do excited SO(3) Einstein-Yang-Mills sphalerons [9, 10].

Acknowledgement We gratefully acknowledge discussions with M. Volkov.

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Mass vs. Coupling

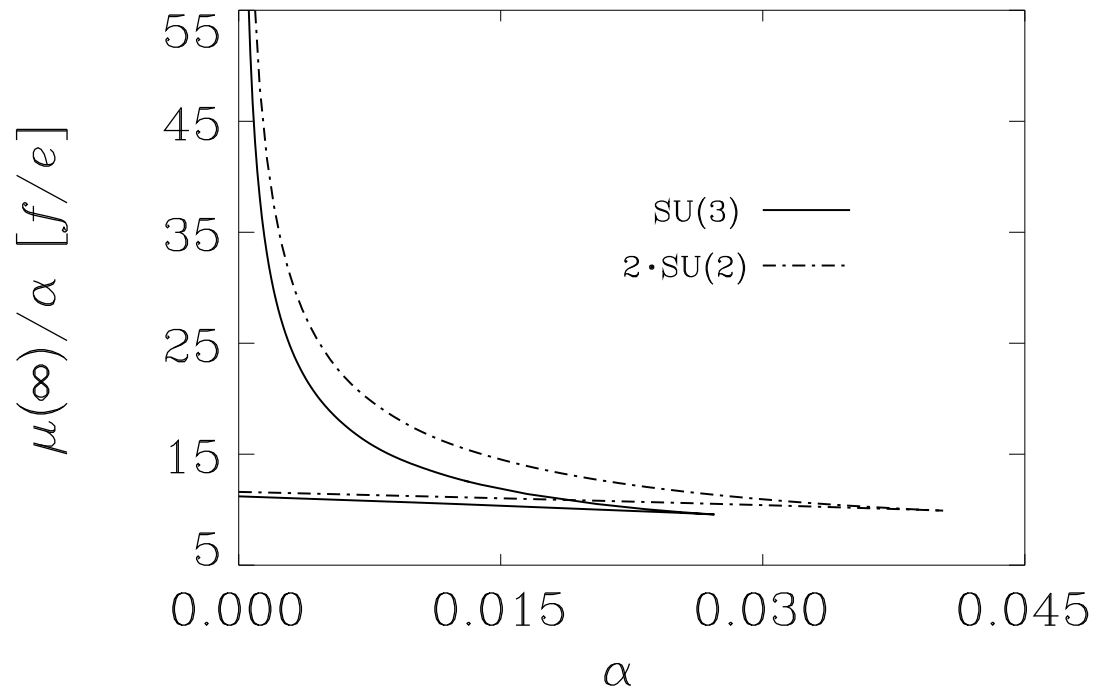


Figure 1: The masses of the regular $B = 2$ SO(3) solutions and twice the masses of the regular $B = 1$ SU(2) solutions are shown as a function of the coupling constant α . Multiplication with 4π gives the ADM mass.

Matter Function ($\alpha = 0.001$)

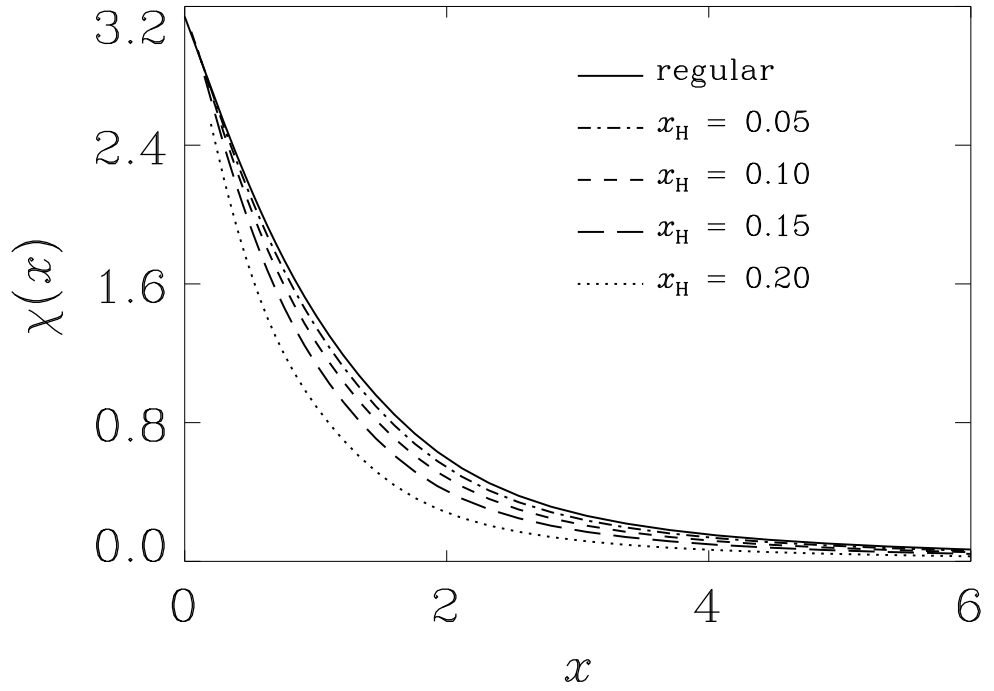


Figure 2: The function $\chi(x)$ is shown for the regular solution and for the black hole solutions with horizons $x_H = 0.05, 0.10, 0.15$ and 0.20 as a function of x for the coupling $\alpha = 0.001$.

Matter Function ($\alpha = 0.001$)

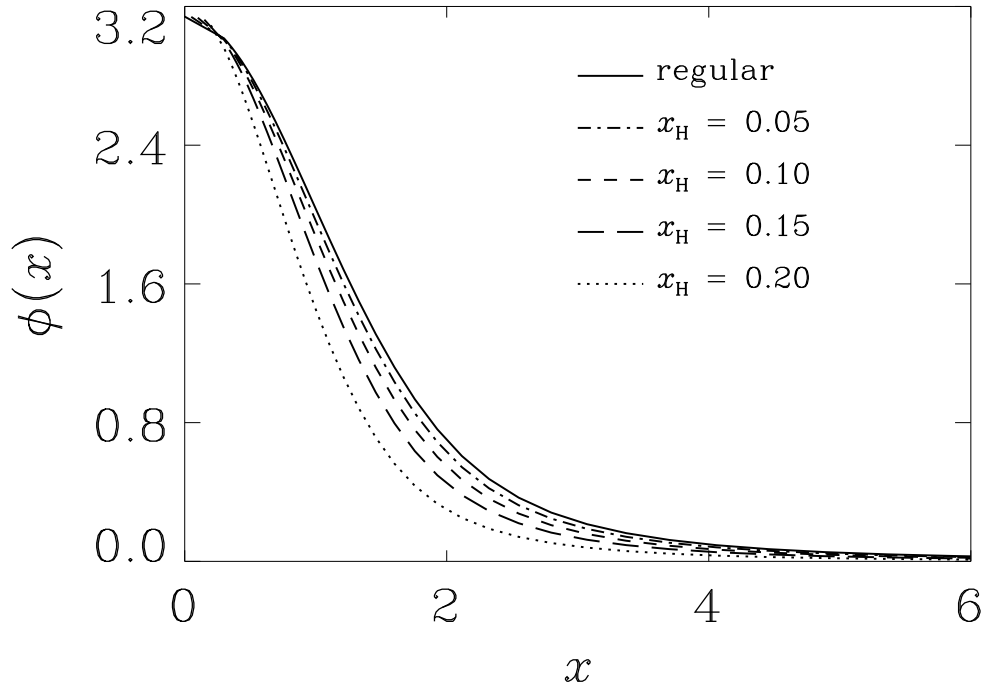


Figure 3: The function $\phi(x)$ is shown for the regular solution and for the black hole solutions with horizons $x_H = 0.05, 0.10, 0.15$ and 0.20 as a function of x for the coupling $\alpha = 0.001$.

Metric Function ($\alpha = 0.001$)

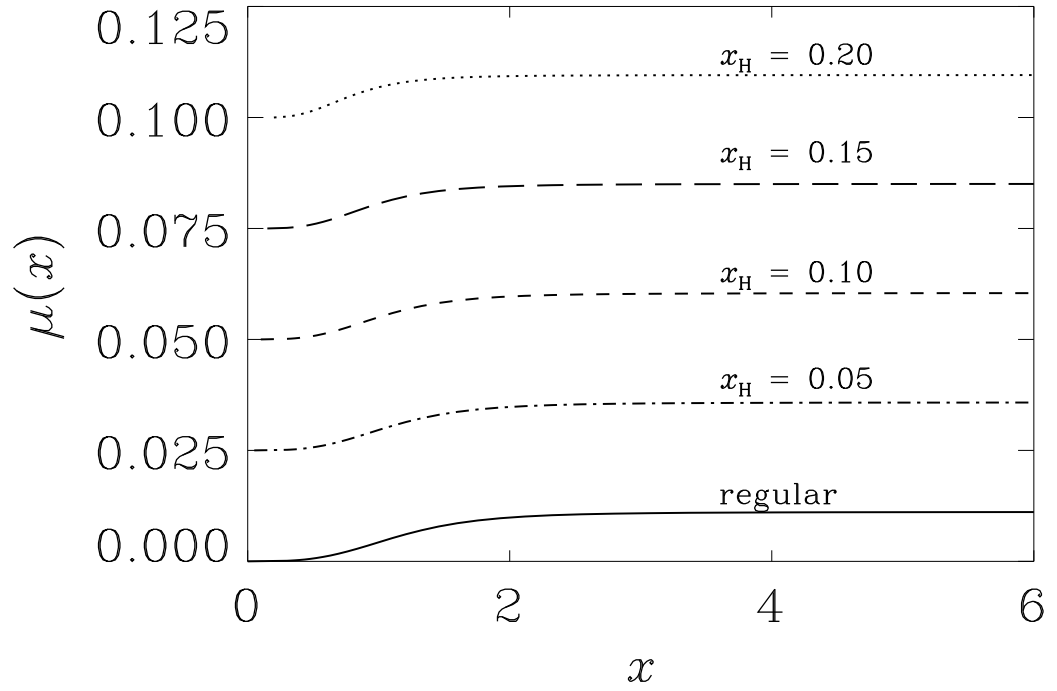


Figure 4: The function $\mu(x)$ is shown for the regular solution and for the black hole solutions with horizons $x_H = 0.05, 0.10, 0.15$ and 0.20 as a function of x for the coupling $\alpha = 0.001$.

Metric Function ($\alpha = 0.001$)

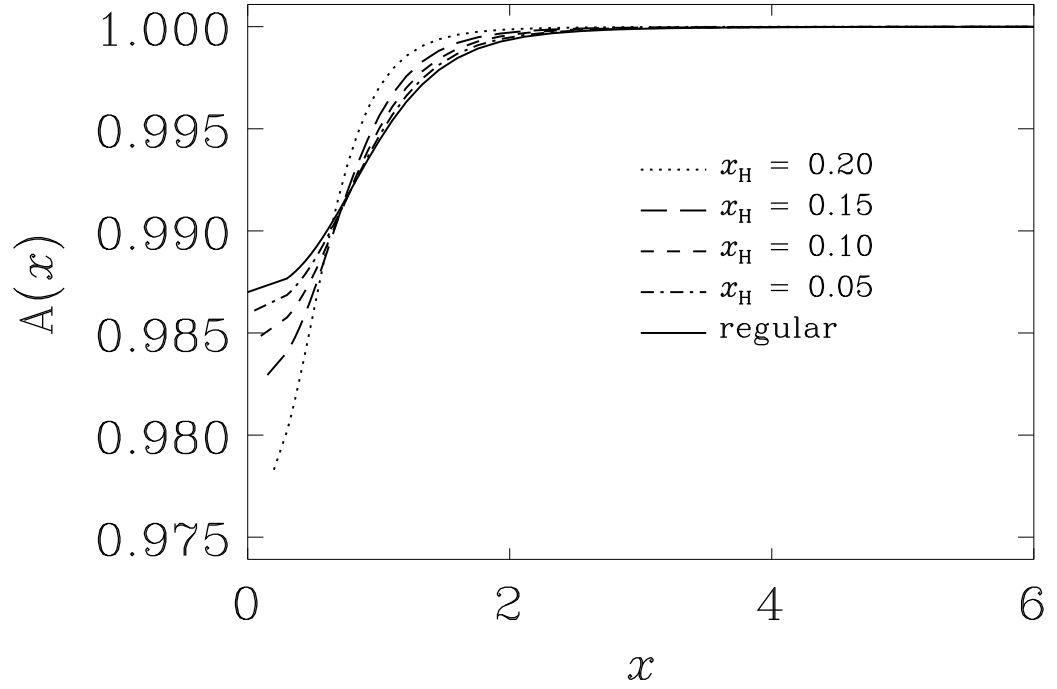


Figure 5: The function $A(x)$ is shown for the regular solution and for the black hole solutions with horizons $x_H = 0.05, 0.10, 0.15$ and 0.20 as a function of x for the coupling $\alpha = 0.001$.

Mass vs. Coupling

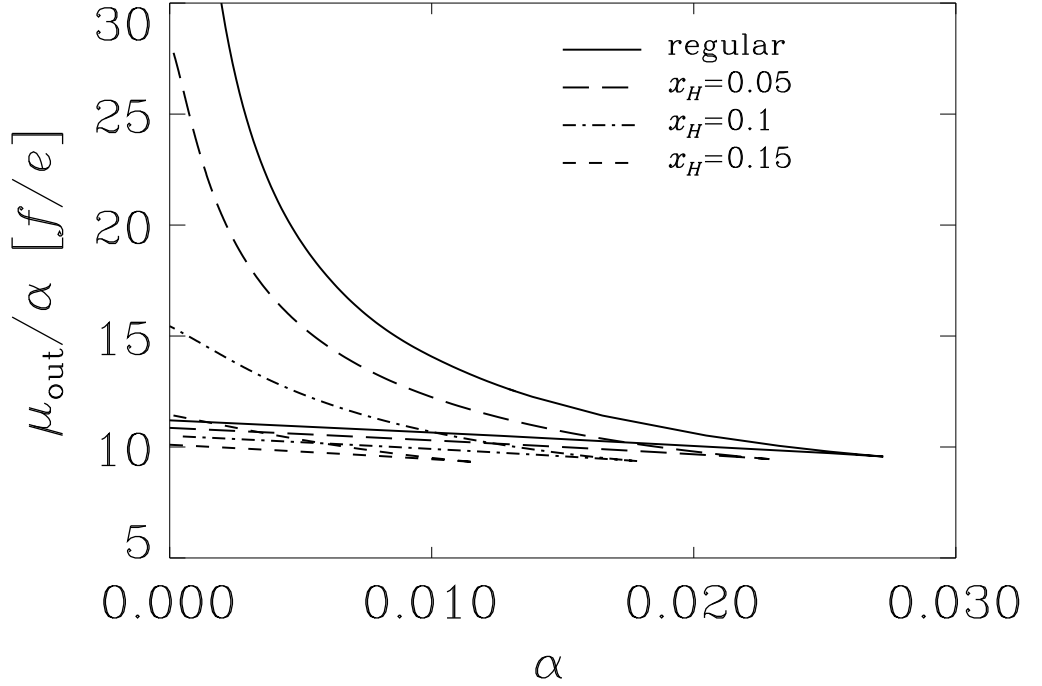


Figure 6: The masses of the regular SO(3) solutions and the mass fractions outside the horizon μ_{out} of the SO(3) black hole solutions are shown as a function of the coupling constant α . When the mass fractions within the horizon, $x_H/2$, are added, multiplication with 4π gives the ADM masses.